REAL TIME DISCRETE FOURIER TRANSFORMS USING CHARGE TRANSFER DEVICES

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ABSTRACT

A real time Discrete Fourier Transform (DFT) device is presented using bucket brigade devices. the chirp - z algorithm is used to compute the DFT. It required premultiplication of the time signal by a complex FM chirp, convolution with a complex FM chirp and postmultiplication by a complex FM chirp. Since the algorithm is complex it is implemented in real arithmetic by four convolvers operating in parallel. A two hundred stage bucket brigade transversal filter performs the convolution required. The signal is tapped with a source follower whose load determines the weighting coefficient. The computation time of the device is proportional to the number of samples, N, in the time signal. The computation time of the Fast Fourier Transform (FFT) in N $\log_2 N$ if N is a power of 2. The device is thus a factor of $\log_2 N$ faster than the FFT. It is also a low power, low weight device.

I. INTRODUCTION

Because of their analog nature, charge coupled devices (CCD's) offer the possibility of performing many of the signal processing functions which until now have fallen into the realm of digital filtering.[1] The discrete Fourier transform (DFT) of an electrical signal is one such linear filtering operation which readily lends itself to implementation with CCD's and is discussed in this paper.

The technique of using CCD's to perform the discrete Fourier transform (DFT) of an electrical signal was first proposed by Whitehouse, Speiser, and Means.[2] The technique they have proposed is called the chirp-Z transform (CZT) because the mathematical operations involved multiplication and convolution with chirp signals.[3]

The CZT was first conceived as an algorithm for the DFT on a digital computer. It never gained wide acceptance, however, because the number of digital operations required for its implementation is the same as for the Cooley-Tukey fast Fourier transform (FFT) algoritym. It can be viewed as three distinct operations, all of which can be performed with analog circuitry: (1) premultiplication by a chirp waveform, (2) convolution with a chirp waveform, and (3) post-multiplication by a chirp waveform. These three operations are illustrated in the block diagram of Figure 1.

The DFT of a band limited electrical signal g(t) is found by sampling the signal at N uniformly spaced instants of time to obtain the N points g_n. The DFT is then given by the definition

$$G_n = \sum_{k=o}^{N-1} g_k \exp[-\frac{i2\pi kn}{N}] \qquad (1)$$

The CZT algorithm for evaluation equation (1) is derived by making the substitution

$$2kn = k^2 + n^2 - (n-k)^2 \qquad (2)$$

into equation (1) to get

$$G_n = e^{-i\pi n^2/N} \sum_{k=o}^{N-1} e^{i\pi(n-k)^2/N} e^{-i\pi k^2/N} g_k \qquad (3)$$

Equation (3) suggests clearly the hardware implementation illustrated in Figure 1. This algorithm offers unique advantages when implemented using CCD transversal filters because all the processing can be performed in an analog format.

Section II of this paper gives preliminary experimental verification of the CZT concept. The demonstration is only approximate but it illustrates the simplicity of the approach. Section III describes the exact CZT implementations. Section IV concludes with the discussion of applications.

II. PRELIMINARY EXPERIMENTAL RESULTS

To demonstrate the concept of the CZT, the simplified system shown at the top of Figure 2 was implemented for approximating the power density spectrum. The filters used in this demonstration have impulse responses which approximate the functions.

$$h(t) = \cos \mu t^2 \quad -\frac{T_d}{2} < t < \frac{T_d}{2} \qquad (4)$$

for the COS filter and

$$g(t) = \sin \mu t^2 \quad -\frac{T_d}{2} < t < \frac{T_d}{2} \qquad (5)$$

for the SIN filter. They are bucket brigade device (BBD) filters which were developed for a spread-spectrum receiver.[4,5,6,7] The impulse responses and correlation responses of these filters are given in Figure 3.

In the demonstration, the chirp signal shown in Figure 2 chirps from 97.5 kHz to 102.5 kHz in 5 msec. and gives the approximate power density spectrum across this frequency band. The operation of the system can be understood in the following way. If the input signal is a single frequency sinusoid of frequency f_s, then the output of the mixer has a difference component which chirps from f_s - 97.5 kHz to f_s -102.5 kHz and a sum component which is filtered out in a low-pass filter after mixing (not shown in Figure 2). The filters are clocked at 20 kHz and have impulse responses which chirp from -5 kHz to +5 kHz in 10 msec.

The convolution operation is indicated schematically at the bottom of Figure 2. The frequency of the filter impulse response is indicated by the line extending from -5 kHz to +5 kHz. The input signals are reversed in time as required by the convolution operation, and are shown for four values of f_s equally spaced across the chirp band: 97.5 kHz, 99.17 kHz, 100.83 kHz and 102.5 kHz. A correlation peak in the filter output occurs when the frequency of the time reversed input signal "lines up" with the frequency of the impulse response. At the time instant shown, the 97.5 kHz signal lines up with the filter and gives a correlation peak. Higher frequency signals result in correlation peaks which occur later in time by the amount Δt given by

$$\Delta t = \frac{\mu}{\pi} (f_s - 97.5 \text{ kHz}) \qquad (6)$$

where $\frac{\mu}{\pi}$ is the slope of the frequency vs time curve as can be seen from eqs. (4) and (5).

Two filters are used to perform in-phase and quadrature processing, and the output signal which results from squaring and summing the filter outputs closely approximates the power density spectrum of the input signal across the 97.5 kHz to 102.5 kHz band.

The system of Figure 2 does not give the exact power density spectrum. There are small spurious sidelobes due to an approximation made that an up chirp is orthoganal to a down chirp. The system output also has a dependence upon the phase of the input signal. This dependence can be reduced by using filters of larger time-bandwidth product or by implementing the exact in-phase and quadrature processing to be discussed in Section III.

The actual operation of the system is illustrated in the series of photographs shown in Figure 4. The input is a single frequency sinusoid whose frequency takes on the values indicated on Figure 2, and the output shows a distinct correlation peak whose position scales uniformly with input frequency. The time axis calibration is 1.67 msec/cm, and the correlation peak shifts by 1 cm each time the input frequency increases by 1.67 kHz ($\mu/\pi = 10^6$ sec^{-2}). The correlation peaks should all be of equal amplitude. However, because of the above mentioned dependence on input phase, there is a small random veriation from one photograph to the next.

III. FOURIER TRANSFORM ANALYSIS

The exact DFT can be implemented by the hardware shown in Figure 5. The complex chirp Z algorithm as defined in equation (3) is implemented by real hardware operating in parallel. The transversal filters discussed in Section II can implement a cosine transform[8] which is a subset of the DFT. In order to illustrate this it is first necessary to discuss the symmetries involved in Fourier transforms.

If the input time signal g(t) is real, as it always is in signal processing applications, then the output Fourier transforms have the property that

$$G_k = G^*_{-k} \tag{7}$$

This is an awkward symmetry to make us of in reducing data handling requirements. However, if the input data is real and symmetric, then the output Fourier transform is also real and symmetric.

This symmetric transform is sometimes referred to in the literature as the cosine transform. It is not uniquely defined and different authors have defined slightly different cosine transforms. There are two implicit symmetrizations possible on the input data: (1) The data set of length N is reflected about a data point. This will result in a data set of length 2N-1; (2) The input data set of length N is reflected so that the resultant data set is of length 2N. An example should clarify these two cases.

1) Original data set: [1,2,3,3,4]
   Symmetrized data set: [4,3,3,2,1,2,3,3,4]

2) Original data set: [1,2,3,3,4]
   Symmetrized data set: [4,3,3,2,1,1,2,3,3,4]

The Naval Undersea Center is interested in implementing the cosine transform because it is involved in a program of television bandwidth reduction. The ordinary discrete Fourier transform has several faults. Since it is really a Fourier transform of a periodically repeated signal, it may have artificial discontinuities at the boundaries. If the signal is made symmetric then the discontinuities disappear and the Fourier transform will thus converge more rapidly. It will be shown later that making the signal symmetric does not take any hardware. In fact it reduces the amount of hardware required to perform a Fourier Transform.

In order to demonstrate this it is advantageous to go through several stages of analysis. Consider a continuous Fourier transform of a finite time signal g(t)

$$G(\omega) = \int_0^T g(t)e^{-i\omega t}\,dt \tag{8}$$

Define the symmetrized time signal as

$$\hat{g}(t) \equiv \begin{cases} g(t) & o \le t \le T \\ g(-t) & -T \le t \le o \end{cases} \tag{9}$$

Then the Fourier transform of $\hat{g}(t)$ is

$$\hat{G}(\omega) = \int_{-T}^{T} \hat{g}(t)e^{-i\omega t}\,dt \tag{10}$$

Since $\hat{g}(t)$ is real and symmetric then $\hat{G}(\omega)$ is real and symmetric and thus

$$\begin{aligned}\hat{G}(\omega) &= \int_{-T}^{T} \hat{g}(t)\cos\omega t\,dt \\ &= 2\int_0^T g(t)\cos\omega t\,dt\end{aligned} \tag{11}$$

For future comparison it is convenient to write this as

$$\hat{G}(\omega) = 2\,\mathrm{Re}\int_0^T g(t)e^{-i\omega t}\,dt \tag{12}$$

Notice that the Fourier transform of the symmetric time signal $\hat{g}(t)$ has been derived by only working with the non-symmetric time function g(t) and that only an integral of length T was required.

Let us consider now a sampled signal g_n of N points and its discrete Fourier transform. Define the symmetric function

$$\hat{g}_n \equiv \begin{cases} g_{n-1} & n \ge 1 \\ g_{-n} & n \le o \end{cases} \qquad -N+1 \le n \le N \tag{13}$$

This function can be looked on as symmetric about n = 1/2. Its Fourier transform is thus

$$\hat{G}_k = \sum_{n=-N+1}^{N} \hat{g}_n\, e^{\frac{-2\pi i k(n-1/2)}{2N}} \qquad -N+1 \le k \le N \tag{14}$$

Since $\hat{g}_n$ is symmetric, then Im $\hat{G}_k = 0$ and $\hat{G}_k = \hat{G}_{-k+1}$ and it is only necessary to compute half the set.

$$\hat{G}_k = \sum_{n=N+1}^{N} \hat{g}_n\, e^{\frac{-2\pi i k(n-1/2)}{2N}} \qquad 1 \le k \le N \tag{15}$$

By rearranging the summing order and using the fact that $\hat{g}_n = g_{n-1}$ for $n \ge 1$, this can be shown to be

$$\hat{G}_k = 2\sum_{n=o}^{N-1} g_n \cos\left[\frac{2\pi k(n+1/2)}{2N}\right] \quad k = 1, N \tag{16}$$

This can be written in the form

$$\hat{G}_k = 2 \text{ Re} \sum_{n=o}^{N-1} g_n \, e^{\frac{2\pi ik(n+1/2)}{2N}} \quad k = 1, N \quad (17)$$

Notice that only the original sampled time function $\{g_n\}$ is used and that the sum is only over N points. Equation 17 can be implemented by a chirp Z transform by substituting the identity.

$$2k(n+1/2) = k^2 + (n+1/2)^2 - (k-(n+1/2))^2 \quad (18)$$

This gives

$$\hat{G}_k = 2 \text{ Re } e^{\frac{-i\pi k^2}{2N}} \sum_{n=o}^{N-1} e^{+i\pi(k-(n+1/2))^2/2N} * e^{-i\pi(n+1/2)^2/2N} \, g_n \quad (19)$$

This cosine transform can be implemented by the bucket brigade devices by letting N=199 except for a factor of 2 in the total length of the filter. This implies that only the first 100 cosine transform coefficients will be computed and that the data is defined to be equal to zero after n=100.

IV. APPLICATIONS

The CZT algorithm is conceptually a real time algorithm for a sampled data system. The set of points $\{g_n\}$ can be considered as a time sampled function of a continuous function g(t) of length T. The transversal filter of the chirp Z transform will have a basic clock frequency equal to the sampling frequency. It has 2N (or 2N-1) stages so it is approximately twice as long as the input signal. A time T must pass before the set $\{g_n\}$ fills the first half of the transversal filter. Thereafter, one DFT doeffieient is computed for every time step. This is what is meant by a real time device, i.e. for a data sequency of length N it takes N clock periods to compute the DFT.

By this definition the chirp Z algorithm can be compared with other algorithms. The basic algorithm as defined in equation (1) takes N^2 steps to compute. If N is a power of 2, then the Cooley-Tukey algorithm called the Fast Fourier Transform (FFT) requires $N \log_2 N$ steps to compute. The transversal filter implementation of the chirp Z algorithm achieves its high speed not be reducing the number of multiplies like the FFT but by putting the equivalent of N parallel multipliers all on the same chip. This reduces the number of computational steps from N^2 to N.

A real time Fourier transform device will find many applications in radar, sonar and pattern recognition. One of the most promising is in bandwidth reduction of television.[9]

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FIGURES

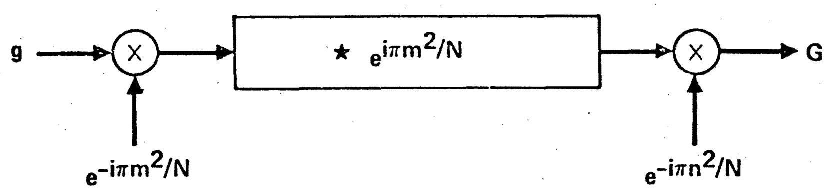

Fig. 1. Chirp Z-transform Implementation of the DFT

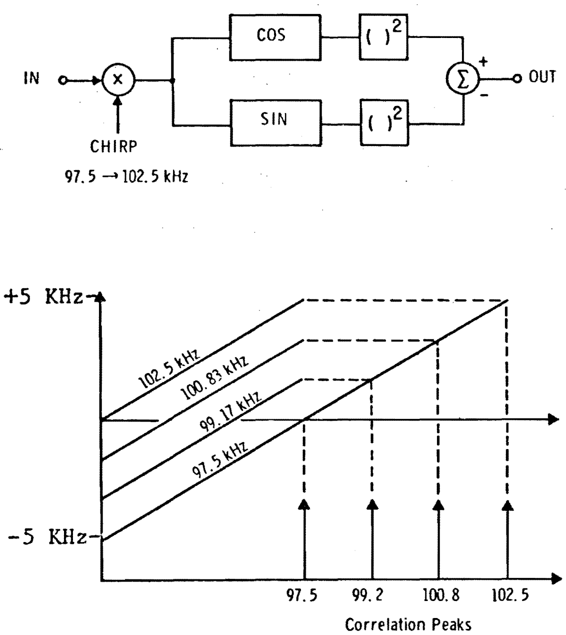

Fig. 2. Top: Block diagram of a configuration for approximating the power density spectrum. The filters marked COS and SIN are clocked at 20 kHz so that their impulse responses chirp from -5 kHz to +5 kHz. A low pass filter which is not shown follows the mixing operation and eliminates the sum frequency components. Bottom: Schematic of the convolution operation performed within the filters. (See text)

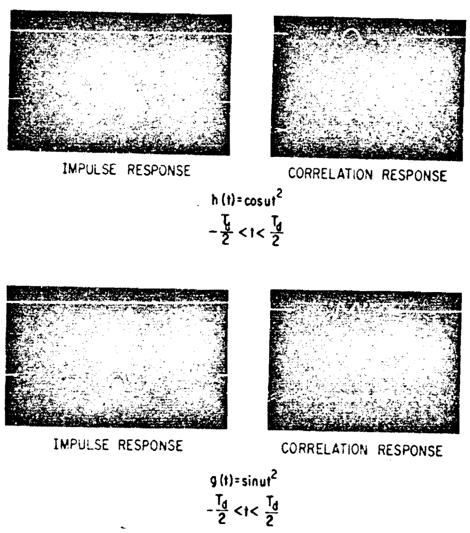

Fig. 3 The impulse response and correlation response of the two filters required for the system of Fig. 2. These filters are each 200 stages long and are both integrated in a single BBD IC.

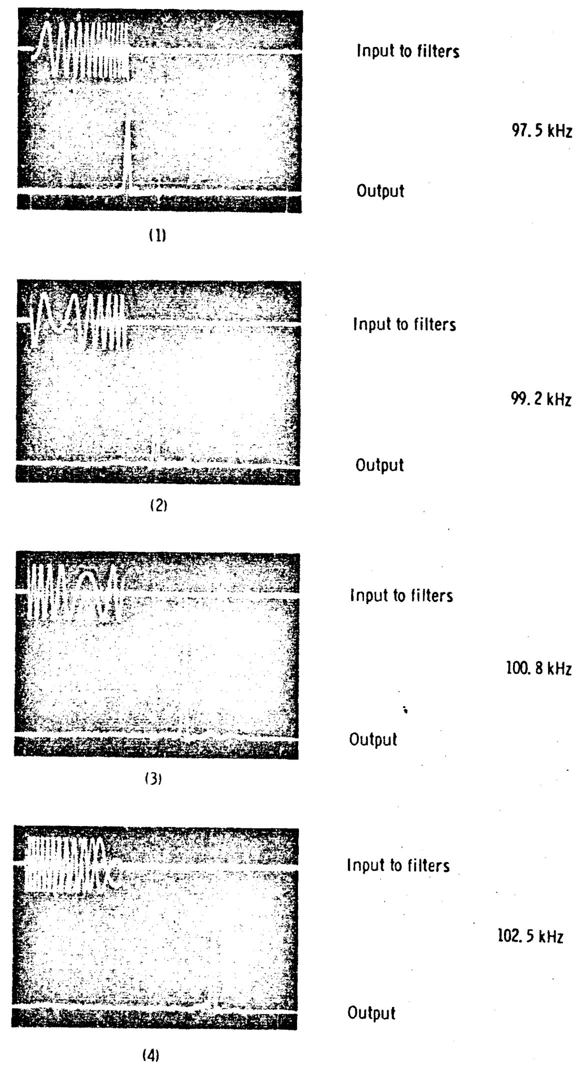

Fig. 4. Power density spectrum output for sinusoidal input

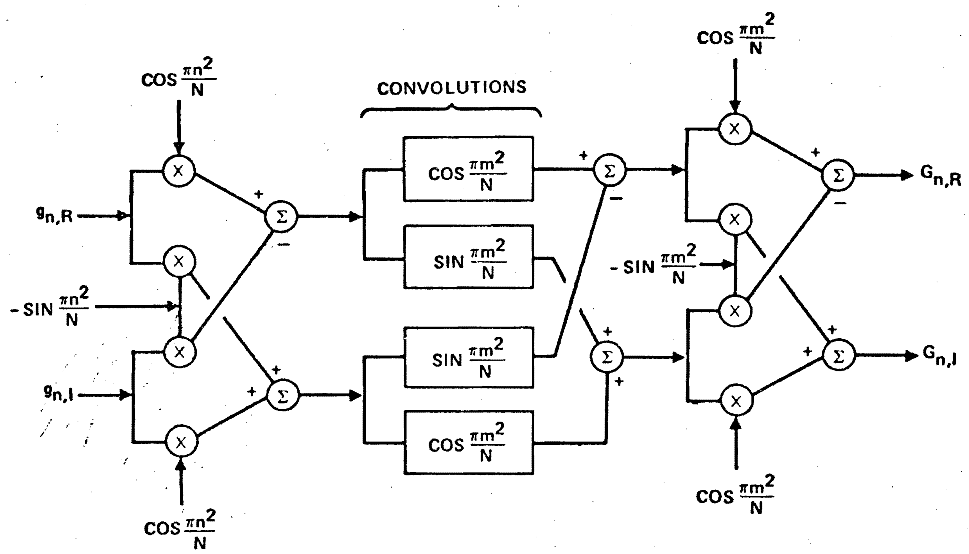

Fig 5 DFT via CZT Algorithm with Parallel Implementation of Complex Arithmetic

REVERSE SIDE BLANK