# Read Noise Distribution Measurement and Modeling of CMOS Image Sensors

Boyd Fowler, Clemenz Portmann, Lele Wang and Steve Tran

Google Inc. 1600 Amphitheater Parkway, Mountain View CA 94043 USA

#### **Abstract**

Noise sets the fundamental limit on image sensor performance. Optimizing image sensor performance requires not only reduction in the average or median noise level for each pixel, but it also requires minimizing the spread of the noise distribution across the image sensor. In this paper, we present measured noise distributions from 13 different CMOS image sensors across multiple manufacturers and pixel generations down to 1.12um. We also show that the heaviness of the read noise distribution tails are not improving as pixel pitch scales.

#### Introduction

Noise sets the fundamental limit on image sensor performance. Optimizing image sensor performance requires not only reduction in the average or median noise level for each pixel, but it also requires minimizing the spread of the noise distribution across the image sensor. Continuous consumer demand for higher resolution and smaller optical format mobile cameras, has motivated image sensor companies to continue developing innovative technologies to shrink pixels [1]. Currently, the smallest pixel in production has a pitch of approximately 1.1um. In this paper, we present measured noise distributions from multiple image sensor manufactures across multiple pixel generations down to 1.1um. In addition, we show how the models described in [2] can be used to understand how cumulative read noise distributions differ between across vendors and process generations. Then we draw conclusions about how read noise distributions are expected to scale in future generations of CMOS image sensors.

## **Modeling**

Equation 1 is the model used to simulate the read noise distributions in [1]. In this section of the paper we will describe how this model can be used to better understand read noise distributions.

$$\sigma_k^2 = c_{fd}^2 \int_0^\infty \left( \frac{4kT}{G_k} + \sum_{i=1}^{N_k} \left( \frac{\gamma_{i,k} X_k^{\alpha_k}}{1 + (f/F_{i,k})^2} \right) \right) H(f) df (1)$$

In (1) the behavior of the read noise distribution's tail is determined by the factor  $X_k^{\alpha_k}$ . Moreover, the extent of the long tail is determined by the magnitude of  $\alpha_k$ . The larger  $\alpha$  is, the longer the read noise tail distribution. This can be seen by simplifying (1) to the following

$$\sigma^2 = \theta + \xi X^{\alpha}$$
, (2)

where  $\sigma^2$ ,  $\theta$ ,  $\xi$ , X and  $\alpha$  are random variables. Assuming that  $\theta$  and  $\xi$  have very narrow distributions in comparison to X,  $\alpha$  we can approximately them as constants. In addition, if we assume that  $\alpha$  is a constant, then we can determine the cumulate distribution of  $\sigma^2$  as a function of the distribution of X. In [1] X has an exponential distribution with parameter 1. Therefore its cumulation distribution function is  $F(x) = 1 - e^{-x}$ . This implies that the cumulative distribution of  $\sigma^2$  is

$$F(\sigma^2) = 1 - e^{-\sqrt[\alpha]{(\sigma^2 - \theta)/\xi}}, (3)$$

and with a bit of algebra on (3), we find that

$$\alpha \log(-\log(1 - F(\sigma^2))) + \log(\xi) = \log(\sigma^2 - \theta). (4)$$

Now if we take the derivative of equation 4 such that

$$\alpha = \frac{\partial \log(\sigma^2 - \theta)}{\partial \log(-\log(1 - F(\sigma^2)))}, (5)$$

we can use (5) to investigate how  $\alpha$  varies as a function of read noise and changes from sensor to sensor. Note that for  $\alpha = 0$  the read noise distribution is close to Gaussian, and for  $\alpha >> 0$  the read noise distribution has a long tail. Although this estimate of  $\alpha$  is poor approximation near the 50th percentile region, it is a reasonable approximation above the 99th percentile.

It is advantageous to obtain a single statistic to help compare the relative heaviness of a distribution's tail. Just such a statistic is

$$HDT = \frac{-\int_{0}^{\infty} \sigma \frac{d \log(1 - F(\sigma))}{d\sigma} d\sigma}{\log(N)(\mu_{\sigma} + \pi \sigma_{\sigma})}, (6)$$

where N is the number of pixels used to calculate the cumulative distribution,  $\mu_{\sigma}$  is the expected value of the read noise distribution, and  $\sigma_{\sigma}$  is the standard deviation of the read noise distribution. This statistic is a log weighted expectation of  $\sigma$  normalized by the log of the number of pixels and the mean and standard deviation of the distribution. This causes large outliers to be heavily weighted in the integral. The normalization allows us to compare distributions with dissimilar numbers of pixels and different means and standard deviations. Note that HDT is equal to 1 for Gaussian distributions. Therefore, HDT is a measure of how much the tail of the distribution varies from a Gaussian.

#### Measurements

For each sensor we first measured the photon transfer curve to determine the conversion gain of the sensor. Under dark conditions at  $25\,^{\circ}$ C ambient temperature we collected 30 images with an integration time of 33ms in sequence. These 30 raw images were processed to estimate mean and standard deviation images. Finally the standard deviation image was used to create a cumulative distribution function of the read noise. The estimated cumulative distribution function was used to calculate  $\alpha$  as function of log(read noise). This was performed by taking the derivative of (4) and performing some low pass filtering to create the final plot. The processed data for the 1.12um pixel sensors is shown in Figure 1. The processed data for the 1.4um pixel sensors is shown in Figure 2, and finally the processed data for the 1.75um pixel sensors is shown in Figure 3. Note the in the upper right corner of each plot there is a legend that describes each sensor with a single letter. Table 1 compares the heaviness of the different distribution's tails using the HDT statistic described in the previous section. The final row of the table is the mean divided by the standard deviation of the HDT statistics in each column.

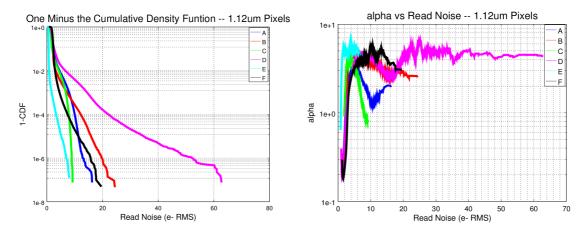


Figure 1: Left: 1-CDF for 1.12um pixel sensors, Right:  $\alpha$  vs read noise for 1.12um pixel sensors.

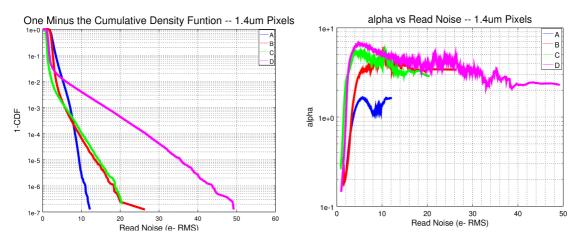


Figure 2: Left: 1-CDF for 1.4um pixel sensors, Right:  $\alpha$  vs read noise for 1.4um pixel sensors.

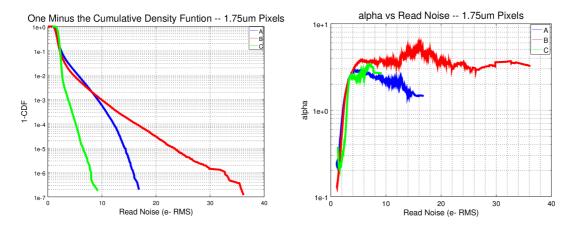


Figure 3: Left: 1–CDF for 1.75um pixel sensors, Right:  $\alpha$  vs read noise for 1.75um pixel sensors.

Sensor Index	1.12um Pixels	1.4um Pixels	1.75um Pixels
A	2.27	1.52	2.14
В	3.41	2.59	3.70
С	1.85	3.81	1.65
D	4.46	4.38	-
E	3.75	-	-
F	2.72	-	-
μ/σ	3.15	2.4	2.33

Table 1: HDT Statistic Comparison.

#### Discussion

The measured data in the last section shows 13 different sensors from 3 different pixel generations across multiple manufactures. The 1–CDF plots are a quick visual comparison of the distributions, but the plots of  $\alpha$  vs log(read noise) show us the characteristics of the distribution tail independent of scale. For example, a sensor with lower read noise may appear to have a reasonable tail distribution, such as sensor E in the 1.12um pixel plot, but further investigation of  $\alpha$  vs log(read noise) shows that sensor E's tail distribution is similar to sensor D. Although the measured data shows significant variation between sensors, it also shows that the read noise tail distributions are similar or possibly worse as pixel pitch scales. Moreover, comparison of the HDT statistics shows that the heaviness of the read noise distribution tail's are not getting better but worse as pixel pitch scales.

### **Conclusions**

We have measured the read noise distributions of the 13 commercially available CMOS image sensors. In addition, we have shown how the read noise model in [2] can be used to understand the read noise tail distribution of various CMOS image sensors. We have also proposed a new single-value statistic to help compare the heaviness of read noise distribution tails. Finally, we have used this statistic to show that the heaviness of the read noise distribution tails are not getting better but worse as pixel pitch scales.

#### References

- [1] H. Tian et al., "Architecture and Development of Next Generation Small BSI Pixels," In *Proceedings of the 2013 International Image Sensor Workshop*, Snowbird Utah, 2013.
- [2] B. Fowler, D. McGrath and P. Bartkovjak, "Read Noise Distribution Modeling for CMOS Image Sensors," In *Proceedings of the 2013 International Image Sensor Workshop*, Snowbird Utah, 2013.