On the covariance and variance in the determination of PTC.

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Photon shotnoise, electron shotnoise

Photons follow Poisson statistics with p the actual number of generated photons with P=*E*(p) the average number of photons, and Pois(p,P) the probability of occurrence of p

$$Pois(p,P)=\frac{e^{-P}\cdot P^{p}}{p!}$$

Given a volume and timeslot a photon either generates an electron-hole pair in it or it doesn't. The generation follows a binominal distribution

$$Bin(n,p,\eta)=\frac{p!}{n!\cdot(p-n)!}\cdot\eta^{n}\cdot(1-\eta)^{p-n}$$

with probability η (=the effective collection efficiency or effective QE), p the number of photons and n the number of electrons generated. The result of the cascaded Poisson and binominal distribution

$$\sum_{p=n}^{\infty}Bin(n,p,\eta)\cdot Pois(p,P)=Pois(n,\eta\cdot P)=Pois(n,N)$$

is again a Poisson distribution with the average number of electrons *E*(n) = N = η *P. Hence the variance (*VAR*) is the same as the average number of electrons, *VAR*(n) = N, the well-known shotnoise. It also implies that when the electrons, which are Poisson distributed, enter another binominal processes (e.g. random crosstalk) the result will be a Poisson distribution again.

> Note on cascading: given P as the average number of photons. The probability that there are p photons generated is Pois(p,P). Those p photons then generate n electron-hole pairs with probability B(n,p, η) but with n<=p. In the case of e.g., electron multiplication n>p one has a Poisson branched process [1] and results differ.

Random crosstalk versus deterministic crosstalk

For simplicity's sake assume a crosstalk between a pixel (pix3) and its left (pix2) and right neighbors (pix4) pixels and vice versa, Table 1. Consider flatfield conditions *E*(n2)=*E*(n3)=*E*(n4)=N and *VAR*(n2)=*VAR*(n3)=*VAR*(n4)=N and *COV*(n3,n2)=*COV*(n3,n4)=*COV*(n2,n4)=0. Given photon-generated-electrons in a slab of silicon, like in BSI, before they are collected in the pixel.

n1	**n2**	**n3**	**n4**	**n5**
Pix1	**Pix2**	**Pix3= central pixel**	**Pix4**	**Pix5**
ε*n2	(1-2ε)*n2	ε*n2		
	ε*n3	(1-2ε)*n3	ε*n3	
		ε*n4	(1-2ε)*n4	ε*n4

Table 1: 1-dim crosstalk with only correlated contributions wrt the central pixel, pix3, recorded.

Random case: assume a probability ε that an electron moves to a neighboring pixel or (1-ε) for moving to the central pixel. Now the average charge in the central pixel is *E*(pix3) = *E*(n2*ε+n3*(1-2ε)+n4*ε)=N and the variance is *VAR*(pix3) = *VAR*(n2*ε+n3*(1-2ε)+n4*ε) = ε*N+(1-2ε)*N+ε*N = N. Use is made of the property that cascading of Poisson, Binominal and Binominal is again Poisson.
Deterministic case: assume that a fixed fraction ε of electrons moves to a neighboring pixel. Now the average charge in the central pixel is *E*(pix3) = *E*(n2*ε+n3*(1-2ε)+n4*ε) = N and variance *VAR*(n2*ε+n3*(1-2ε)+n4*ε) = $ε^2$*N+$(1-2ε)^2$*N+$ε^2$*N =(1-4ε+6$ε^2$)*N. The covariance wrst the crosstalk carrying pixels is *COV*(pix3,pix1+pix2+pix3+pix4+pix5)= *COV*(ε*n2+(1-2ε)*n3+ε*n4, n2+n3+n4)=(

ε*N+(1-2ε)*N+ε*N)=N. Now the covariance has the correct "shotnoise term" for PTC gain calculation [2] and the variance is estimating the "shotnoise" to low.
The difference between random- or deterministic crosstalk shows up in the variance. Average signal level is N in both cases. In the case of a deterministic crosstalk one can expect a deviation for the gain [DN/e] estimated with the variance [2]. Resulting in an estimated gain in [DN/e] which is smaller than the actual gain. All the parameters expressed in electrons are then estimated too high.

PTC and gain [DN/e]
In astronomical imaging one has measured that the PTC (*VAR*(S)/S) [2] can be non-linear even though the light-to-outputsignal is linear (variable integration time at constant illumination) [3,4]. It is assumed to be contributed to charge sharing with neighboring pixels. The crosstalk ε increases linear with the number of electrons in a charge packet, $\varepsilon=\varepsilon_0+\beta*N$. The more electrons the more more sharing, percentwise the crosstalk increases. It is because of the crosstalk ε dependence on the amount of charge N that it became visible in the PTC. One of the solutions in this specific situation was calculating the covariance instead of the variance to arrive at the proper PTC and the gain (DN/e) could be determined correctly over a major part of exposure levels.

Use of autocorrelation and covariance.
To check on the use of the covariance as a possible general approach a couple of measurements have been reworked.
For definition sake the shotnoise image is a difference image between two exposed images taken shortly after another under the same conditions [5]: Shot = Gray1 - Gray2. This image contains only temporal noise, all fixed patterns like FPN, PRNU, Columns, rows are removed.
First, the autocorrelation (*RHO*) was calculated on the shotnoise image (Shot) to check if there was crosstalk and how many neighboring pixels were involved in the crosstalk. The cause for the crosstalk can be anywhere in the signal chain from the photon-generated-electrons to the measurement output.
The algo is: calculate the correlation between Shot and the shifted version of Shot. The shift is variable between -5:+5 pixels in H and V independently. The result is a 11x11 raster of correlation values, Figures 1a and 2a.
Next is the calculation of the covariance. In this paper two implementations for the covariance are used. The algo is: calculate for each exposure level the covariance of a pixel in the shotnoise image (Shot) with the pixel neighbors in a 3x3 kernel:
COV(pixel, sum of all surrounding pixels in 3x3 kernel) this contains the variance including the crosstalk.
COV(pixel, sum of all surrounding pixels in 3x3 kernel with central pixel removed) this contains only the crosstalk. It shows what is "under-the-hood" of the pixel including signal processing.

Figure 1a right: correlation coefficients are noisy and small, in the range of 0.2% - 0.3% for the neighbors. In a helicopter view (Fig 1a left) the correlation is limited to one pixel.
Fig 1b right: variance and covariance PTC coincide.
Fig 1b left: the contribution of the neighboring pixels (red-trace) compared to the variance (blue-trace) is negligible. The black-trace is the measurement threshold for the crosstalk (red-trace) showing no significance for the crosstalk.
Figure 2a right: Positive correlation coefficients in the range of 2%-4% with its neighbors.
Figure 2b right: at medium exposures<2000LSB, the variance and the covariance PTC coincide but for larger exposure values they deviate. The PTC covariance curve is hovering above the PTC variance curve. The slope for the variance-based PTC is 1 and the slope of the covariance PTC based >1. Indicating another type of cross talk is present.
Figure 2b left: the correlated contribution of neighboring pixels is negligible for exposures less than <2000LSB, but increases sharply for exposures >2000LSB. The black-trace is the measurement

threshold for the crosstalk (red-trace) showing significance for exposures>2000LSB. The "under-the-hood" effect clearly visible. Clearly not the effect reported in [3,4] and needs further investigation.

Conclusion

No general use of the covariance possible for an improved determination of the gain [DN/e] through PTC. It needs to be checked for each situation eg through application of the autocorrelation and the covariance with neighboring pixels only.

The autocorrelation of the shotnoise image shows the extend of the crosstalk, if any, both spatially and in amplitude. It does not tell where the cause of the crosstalk is located.

Surprisingly the covariance with the variance term removed shows what is "under the hood of the pixel" and in general the full signal path from pixel (photon generated electrons) to output.

References

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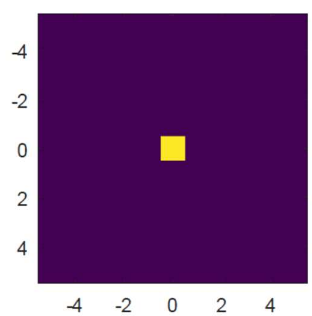

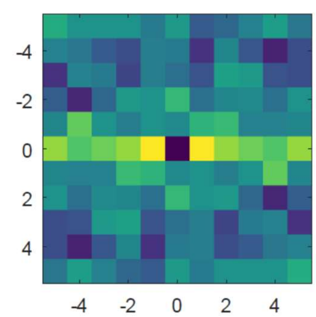

Figure 1a: Surface plots of the autocorrelation of the shotnoise image Shot = (Gray1 – Gray2) at +-5 pixelshift with center at 0,0. **Left:** autoscaled surface plot. The autocorrelation seems only 1 pixel wide, itself, and is 1 at 0,0. **Right:** autoscaled surface plot with the center value "1" removed. The correlation coefficients look random like and with small values. Median value RHO=0.14% and max 0.3%.

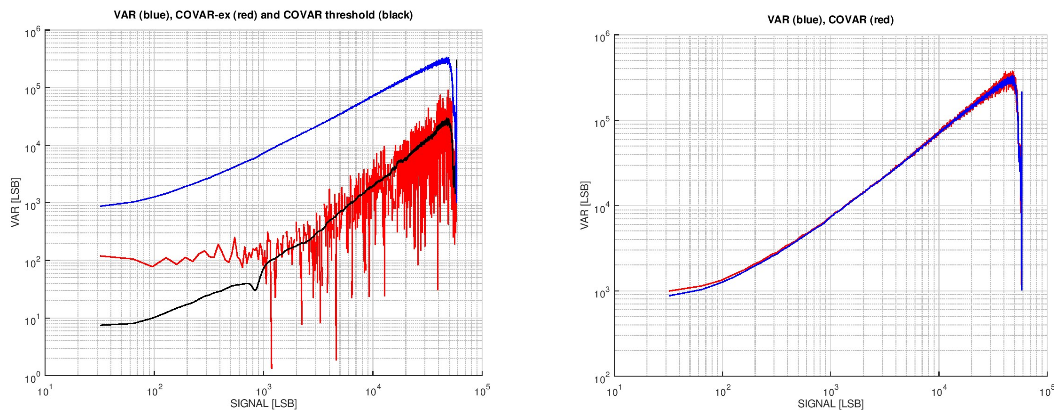

Figure 1b: Left: variance versus signal level. Blue-trace: variance only, Red -trace covariance with variance removed showing the crosstalk only. Black-trace is measurement threshold. **Right:** variance (blue) and covariance (red) in one graph.

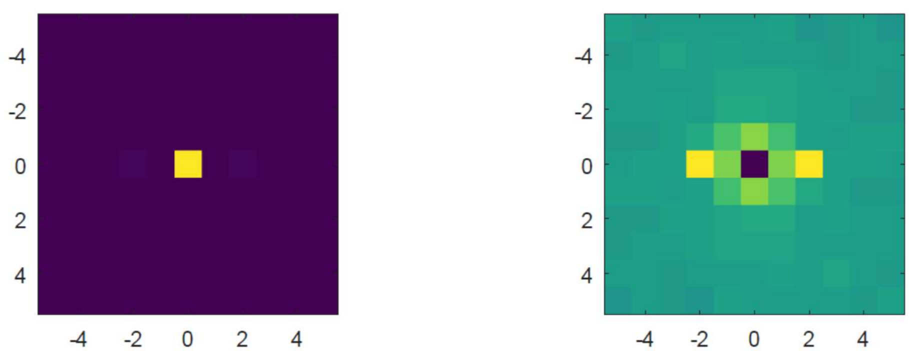

Figure 2a: Surface plots of the autocorrelation of the shotnoise image Shot=(Gray1- Gray2) at +-5 pixelshift, with center at 0,0. **Left:** autoscaled surface plot. The autocorrelation seems only 1 pixel wide, itself, and is 1 at 0,0. **Right**: autoscaled surface plot with the center value "1" removed. The correlation coefficients show a clear crosstalk with neighbor pixels. Median value RHO=2.2% and max 4%.

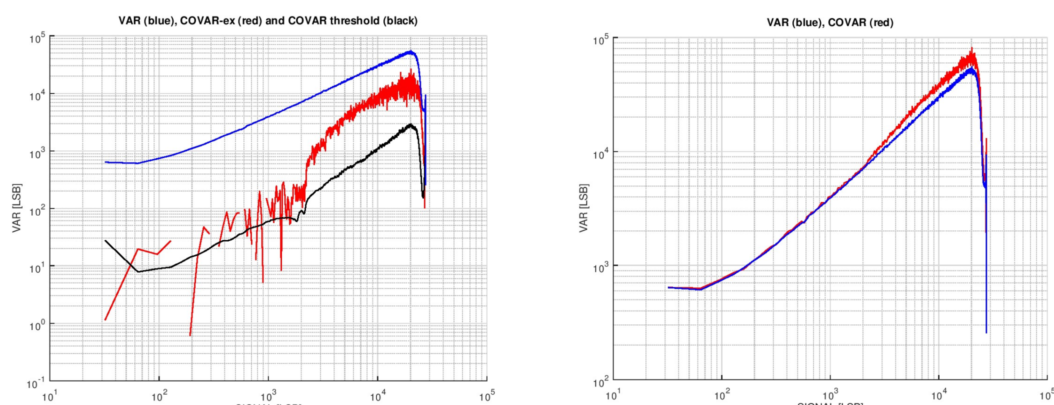

Figure 2b: Left: variance versus signal level. Blue-trace: variance only, Red-trace covariance with variance removed showing the crosstalk only. Black-trace is measurement threshold. **Right:** variance (blue) and covariance (red) in one graph.