Modeling and verification of a photon-counting LiDAR

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Abstract This paper presents a complete model for simulating minimum ranging time with given physical parameters in a LiDAR module. The model has been compared with the measurement results at various stages and good consistency between the theory and experiment has been obtained.

Keywords: LiDAR, single-photon avalanche diode, minimum ranging time, modeling

1. Introduction

In this decade, light detection and ranging (LiDAR) system attracts increasing attentions for their potential uses in the perception layer of driverless vehicle and advanced driving assistance system (ADAS) [1-12]. LiDAR has good image resolution and reliable distance information which is crucial for road safety. The demand of high laser power for long-distance and high-speed ranging makes its module cost not acceptable by the market. To lower the laser power, at the receiver end, using silicon-based CMOS-compatible sensors such as single-photon avalanche diodes (SPADs) is one of approaches in these years as a good candidate for pulsed-mode time-of-flight (ToF) ranging or photon-counting based LiDAR thanks to their excellent photon sensitivity and timing resolution [1,3,6].

2. Minimum ranging time - Tmin

It has been a difficult task to compare or to evaluate LiDAR modules as they are consisted of so many components and could be deployed in different schemes [2,4,5,7]. A core parameter for LiDAR performance is the minimum ranging time for single point measurement, defined as the shortest required time for a distance measurement with an acceptable missing rate. In this work, we proposed an analytical model to quantitatively simulate Tmin with the system physical parameters. In our model, the two links have been established. The first one is between the physical parameters of the LiDAR module and the return photo-counts per pulse taking the data loss due to deadtime of SPAD into account. The second one is the minimum ranging time simulation provided that the laser counts and noise counts, mainly from the background sunlight, are given. With the connected two links, a complete LiDAR model has been proposed in this report.

3. Model verification

The proposed model has been verified experimentally by using our LiDAR module consisted of a 64x128 SPAD chip array, a pulsed 940-nm photonic-crystal surface-emitting laser (PCSEL) driven by a commercial pulse generator, a time-correlated single-photon card (TCSPC), and a two-axis galvo-mirror for scanning and optical alignment. The theoretical return photo-counts at zero deadtime (RP_0) is about 1.5 to 2 times to the experimental one and follows a poissonian distribution. The laser/noise counts inside/outside the target time window, instead, follows a binomial distribution. An excellent consistency between the simulated and measured Tmin has been obtained with near 30 measurement conditions.

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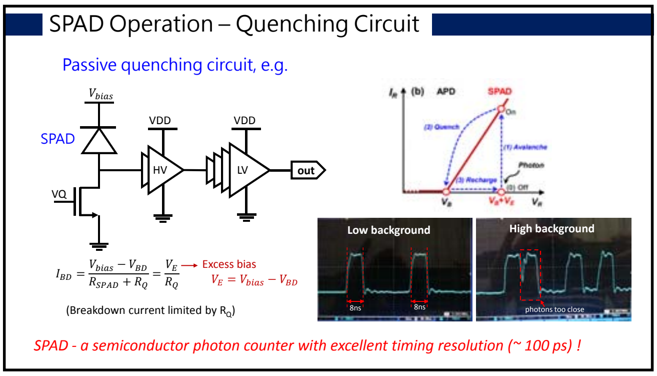


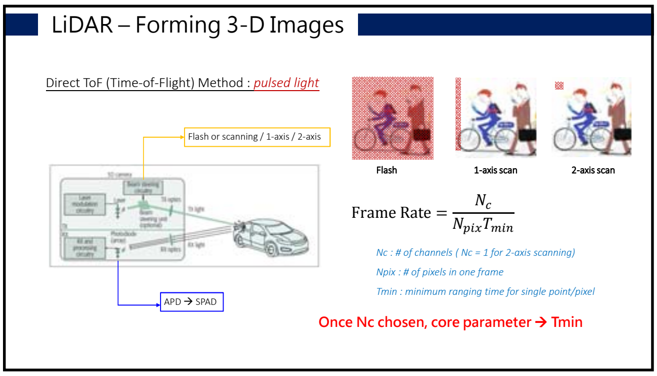
SPAD Operation – Quenching Circuit

Passive quenching circuit, e.g.

$$I_{BD} = \frac{V_{bias} - V_{BD}}{R_{SPAD} + R_Q} = \frac{V_E}{R_Q} \rightarrow \text{Excess bias} \quad V_E = V_{bias} - V_{BD}$$

(Breakdown current limited by R_Q)

SPAD - a semiconductor photon counter with excellent timing resolution (~ 100 ps) !



LiDAR – Forming 3-D Images

Direct ToF (Time-of-Flight) Method : *pulsed light*

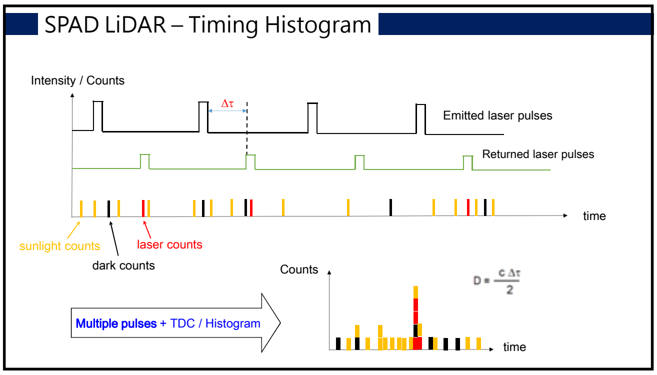
$$\text{Frame Rate} = \frac{N_c}{N_{pix} T_{min}}$$

Nc : # of channels (Nc = 1 for 2-axis scanning)

Npix : # of pixels in one frame

Tmin : minimum ranging time for single point/pixel

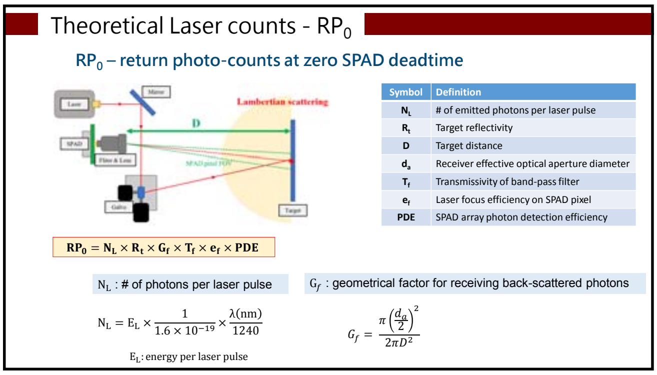
Once Nc chosen, core parameter → Tmin



SPAD LiDAR – Timing Histogram

$$D = \frac{c\,\Delta\tau}{2}$$

Multiple pulses + TDC / Histogram



Theoretical Laser counts - RP_0

RP_0 – return photo-counts at zero SPAD deadtime

Symbol	Definition
N_L	# of emitted photons per laser pulse
R_t	Target reflectivity
D	Target distance
d_a	Receiver effective optical aperture diameter
T_f	Transmissivity of band-pass filter
e_f	Laser focus efficiency on SPAD pixel
PDE	SPAD array photon detection efficiency

$$\mathbf{RP_0 = N_L \times R_t \times G_f \times T_f \times e_f \times PDE}$$

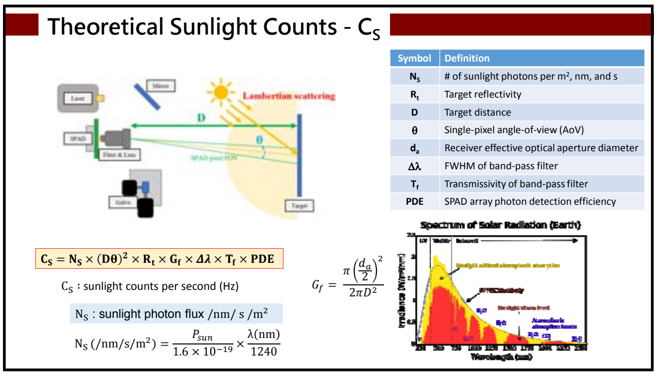
N_L : # of photons per laser pulse

$$N_L = E_L \times \frac{1}{1.6 \times 10^{-19}} \times \frac{\lambda(\text{nm})}{1240}$$

E_L: energy per laser pulse

G_f : geometrical factor for receiving back-scattered photons

$$G_f = \frac{\pi \left(\frac{d_a}{2}\right)^2}{2\pi D^2}$$



Theoretical Sunlight Counts - C_S

Symbol	Definition
N_S	# of sunlight photons per m², nm, and s
R_t	Target reflectivity
D	Target distance
θ	Single-pixel angle-of-view (AoV)
d_a	Receiver effective optical aperture diameter
Δλ	FWHM of band-pass filter
T_f	Transmissivity of band-pass filter
PDE	SPAD array photon detection efficiency

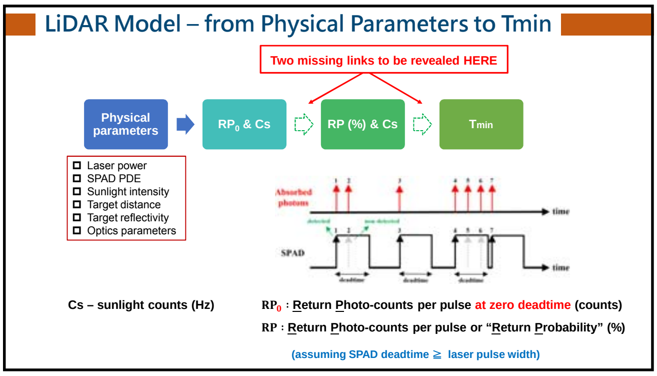
$$\mathbf{C_S = N_S \times (D\theta)^2 \times R_t \times G_f \times \Delta\lambda \times T_f \times PDE}$$

C_S : sunlight counts per second (Hz)

N_S : sunlight photon flux /nm/ s /m²

$$N_S\,(/\text{nm/s/m}^2) = \frac{P_{sun}}{1.6 \times 10^{-19}} \times \frac{\lambda(\text{nm})}{1240}$$

$$G_f = \frac{\pi \left(\frac{d_a}{2}\right)^2}{2\pi D^2}$$



LiDAR Model – from Physical Parameters to Tmin

Two missing links to be revealed HERE

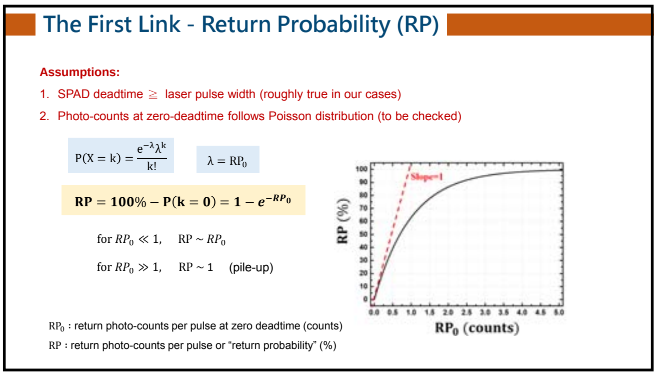
- ☐ Laser power
- ☐ SPAD PDE
- ☐ Sunlight intensity
- ☐ Target distance
- ☐ Target reflectivity
- ☐ Optics parameters

Cs – sunlight counts (Hz)

RP_0 : Return Photo-counts per pulse at zero deadtime (counts)

RP : Return Photo-counts per pulse or "Return Probability" (%)

(assuming SPAD deadtime ≧ laser pulse width)



The First Link - Return Probability (RP)

Assumptions:

1. SPAD deadtime ≧ laser pulse width (roughly true in our cases)
2. Photo-counts at zero-deadtime follows Poisson distribution (to be checked)

$$P(X = k) = \frac{e^{-\lambda}\lambda^{k}}{k!} \qquad \lambda = RP_0$$

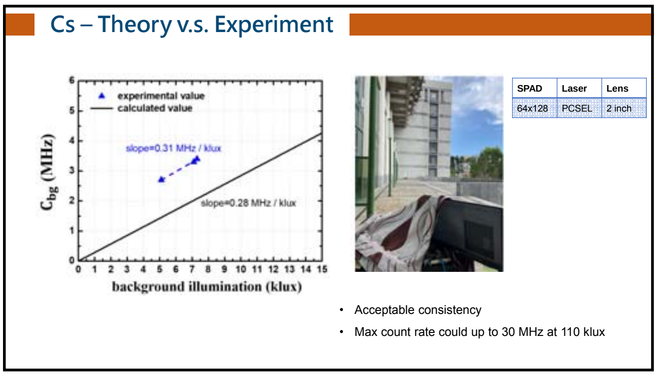
$$\mathbf{RP = 100\% - P(k = 0) = 1 - e^{-RP_0}}$$

for $RP_0 \ll 1$, $\quad RP \sim RP_0$

for $RP_0 \gg 1$, $\quad RP \sim 1$ (pile-up)

RP_0 : return photo-counts per pulse at zero deadtime (counts)

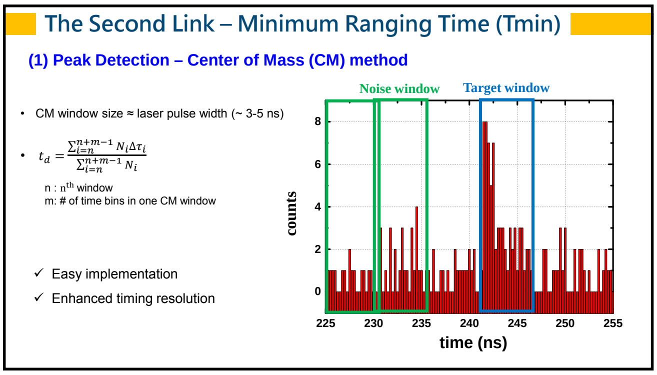
RP : return photo-counts per pulse or “return probability” (%)



Cs – Theory v.s. Experiment

SPAD	Laser	Lens
64x128	PCSEL	2 inch

- Acceptable consistency
- Max count rate could up to 30 MHz at 110 klux



The Second Link – Minimum Ranging Time (Tmin)

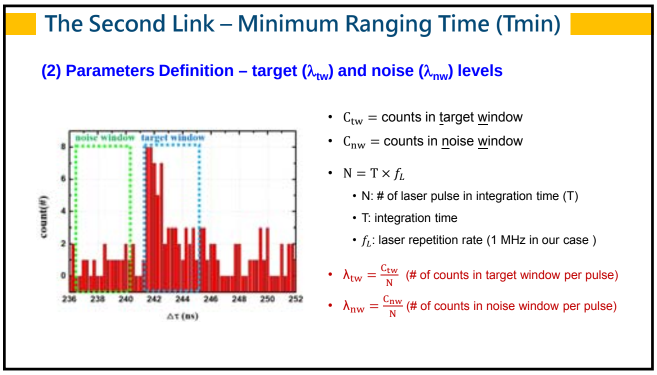
(1) Peak Detection – Center of Mass (CM) method

- CM window size ≈ laser pulse width (~ 3-5 ns)
- $t_d = \frac{\sum_{i=n}^{n+m-1} N_i \Delta\tau_i}{\sum_{i=n}^{n+m-1} N_i}$

  n : n^{th} window
  m: # of time bins in one CM window

✓ Easy implementation

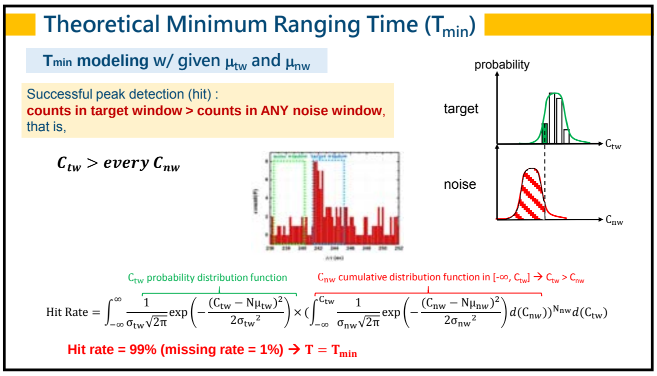
✓ Enhanced timing resolution



The Second Link – Minimum Ranging Time (Tmin)

(2) Parameters Definition – target (λ_{tw}) and noise (λ_{nw}) levels

- C_{tw} = counts in target window
- C_{nw} = counts in noise window
- $N = T \times f_L$
  - N: # of laser pulse in integration time (T)
  - T: integration time
  - f_L: laser repetition rate (1 MHz in our case)
- $\lambda_{tw} = \frac{C_{tw}}{N}$ (# of counts in target window per pulse)
- $\lambda_{nw} = \frac{C_{nw}}{N}$ (# of counts in noise window per pulse)



Theoretical Minimum Ranging Time (T_{min})

T_{min} modeling w/ given μ_{tw} and μ_{nw}

Successful peak detection (hit) :
counts in target window > counts in ANY noise window,
that is,

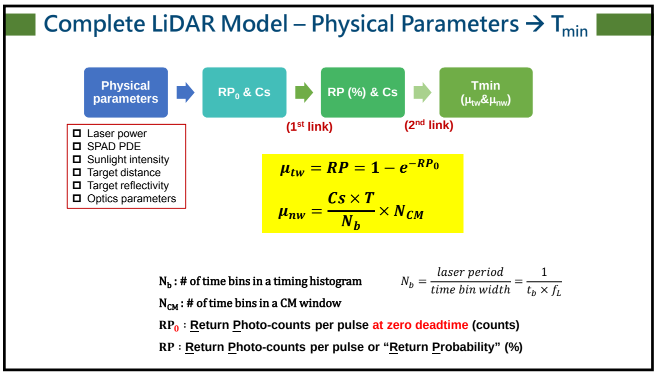
$$C_{tw} > every\ C_{nw}$$

C_{tw} probability distribution function

C_{nw} cumulative distribution function in [-∞, C_{tw}] → $C_{tw} > C_{nw}$

$$\text{Hit Rate} = \int_{-\infty}^{\infty} \underbrace{\frac{1}{\sigma_{tw}\sqrt{2\pi}} \exp\left(-\frac{(C_{tw} - N\mu_{tw})^2}{2{\sigma_{tw}}^2}\right)} \times \underbrace{\left(\int_{-\infty}^{C_{tw}} \frac{1}{\sigma_{nw}\sqrt{2\pi}} \exp\left(-\frac{(C_{nw} - N\mu_{nw})^2}{2{\sigma_{nw}}^2}\right) d(C_{nw})\right)^{N_{nw}}} d(C_{tw})$$

Hit rate = 99% (missing rate = 1%) → $T = T_{min}$



Complete LiDAR Model – Physical Parameters → T_{min}

Physical parameters → RP_0 & Cs → RP (%) & Cs → Tmin (μ_{tw}&μ_{nw})

(1st link) (2nd link)

☐ Laser power
☐ SPAD PDE
☐ Sunlight intensity
☐ Target distance
☐ Target reflectivity
☐ Optics parameters

$$\mu_{tw} = RP = 1 - e^{-RP_0}$$

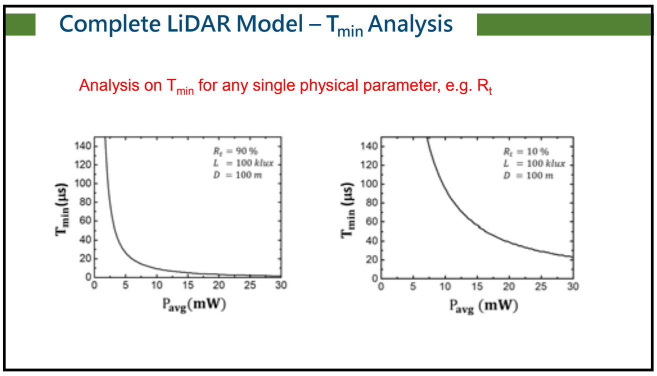
$$\mu_{nw} = \frac{Cs \times T}{N_b} \times N_{CM}$$

N_b : # of time bins in a timing histogram $\qquad N_b = \frac{laser\ period}{time\ bin\ width} = \frac{1}{t_b \times f_L}$

N_{CM} : # of time bins in a CM window

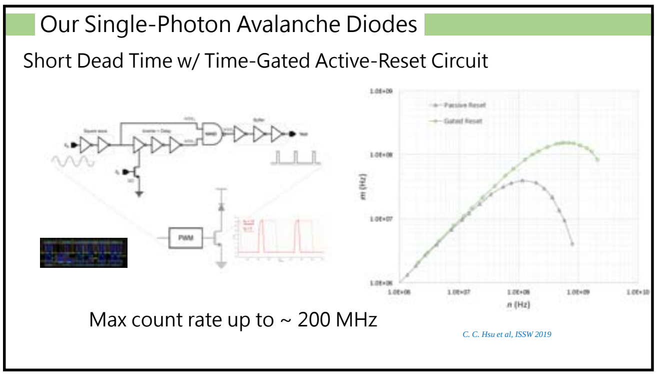
RP_0 : Return Photo-counts per pulse at zero deadtime (counts)

RP : Return Photo-counts per pulse or “Return Probability” (%)



Complete LiDAR Model – T_{min} Analysis

Analysis on T_{min} for any single physical parameter, e.g. R_t

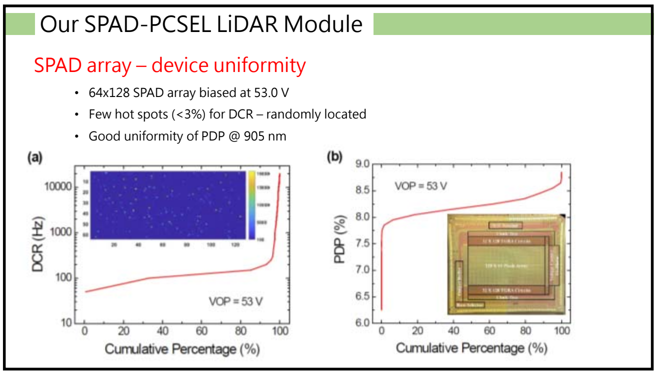


Our Single-Photon Avalanche Diodes

Short Dead Time w/ Time-Gated Active-Reset Circuit

Max count rate up to ~ 200 MHz

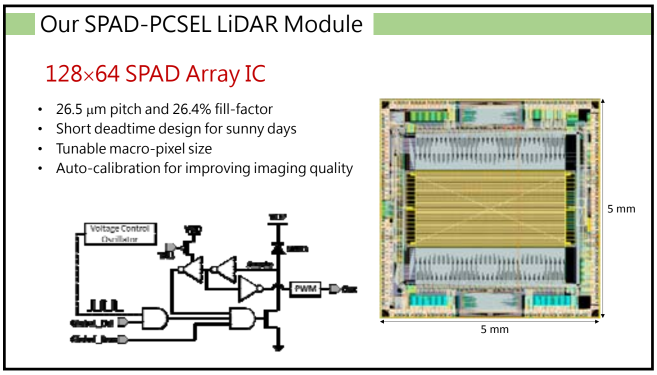
C. C. Hsu et al, ISSW 2019



Our SPAD-PCSEL LiDAR Module

SPAD array – device uniformity

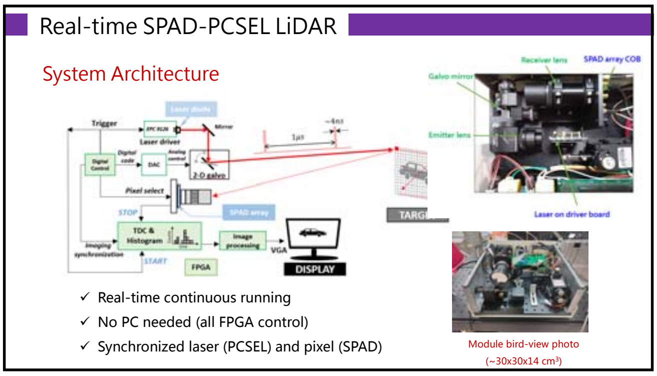
- 64x128 SPAD array biased at 53.0 V
- Few hot spots (<3%) for DCR – randomly located
- Good uniformity of PDP @ 905 nm



Our SPAD-PCSEL LiDAR Module

128×64 SPAD Array IC

- 26.5 μm pitch and 26.4% fill-factor
- Short deadtime design for sunny days
- Tunable macro-pixel size
- Auto-calibration for improving imaging quality

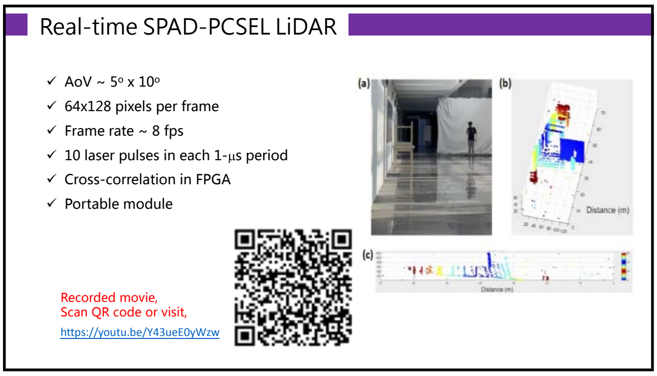


Real-time SPAD-PCSEL LiDAR

System Architecture

- ✓ Real-time continuous running
- ✓ No PC needed (all FPGA control)
- ✓ Synchronized laser (PCSEL) and pixel (SPAD)

Module bird-view photo
(~30x30x14 cm^3)



Real-time SPAD-PCSEL LiDAR

- ✓ AoV ~ 5° x 10°
- ✓ 64x128 pixels per frame
- ✓ Frame rate ~ 8 fps
- ✓ 10 laser pulses in each 1-μs period
- ✓ Cross-correlation in FPGA
- ✓ Portable module

Recorded movie,
Scan QR code or visit,
https://youtu.be/Y43ueE0yWzw