Noise reduction based on quantization-aware multiple image averaging

Seishi Takamura
Faculty of Computer and Information Sciences, Hosei University
3-7-2 Kajino-cho, Koganei-shi, Tokyo 184-8584, Japan
E-mail: takamura@ieee.org

Abstract The process of reducing noise by digitally capturing a still scene many times and applying averaging is widely used, and is useful for increasing the number of tones, improving image quality, and improving coding efficiency. This paper discusses the optical shot noise, which is the dominant and unavoidable noise superimposed on images, and shows that additive averaging may not approach the true value under certain conditions. We also show that the MSE can be reduced to 1/10 to 1/40 of the true value by applying a correction based on theoretical values to conventional averaging, and that even a 10-bit system can sense up to about 1037, which is beyond the upper limit of 1023, by taking advantage of the existence of the optical shot noise.

Keywords: noise reduction, additive averaging, optical shot noise, Poisson distribution

1. Introduction

The performance of image sensors has been increasing in recent years due to technological innovations, and in CMOS image sensors, the reduction of dark current noise and readout noise, two of the three major noise factors in image sensors, has been affirmatively resolved[1], leaving only optical shot noise as the remaining noise factor.

The method of obtaining an additive average of multiple images of a still scene (hereafter referred to as the *additive averaging method*) is widely used as a method to efficiently reduce noise. The additive averaging method is very effective in reducing noise that is independent for each image capture, and it also contributes to increasing the number of gradations and improving coding efficiency.

In this paper, we show that in the current standard image sensing method, which measures the amount of light assumed to have a certain true value and outputs it as a digital value, the additive averaging method may not approach the true value no matter how many images are taken, and that the average value can be corrected based on an optical shot noise model to approach the true value. Furthermore, by applying a correction based on the optical shot noise model to the averaged value, the true value can be approached, and brightness exceeding the upper limit of possible digital values can also be sensed.

2. Additive averaging, quantization and optical shot noise

2.1 Effect of quantization on additive averaging

To investigate the quantitative relationship between digital imaging and the additive averaging method, we add a noise which follows $N(0,\sigma^2)$ to a uniformly distributed value x_t (i.e., true value) and quantize (i.e., round) it to obtain an integer value x_D. We repeat this process F times. Fig. 1 shows the relation of the mean squared error (MSE) between x_t and the additive average of x_D according to the number of F. When $\sigma > 0.5$ or so, the MSE steadily decreases. However, when $\sigma < 0.4$, the MSE does not decrease and approaches a constant value. The additive averaging method is inherently effective, but this is not the case when the noise is relatively small and the system contains quantization.

2.2 Lightness dependence of optical shot noise

Optical shot noise is caused by temporal fluctuations in the number of photons that come to the sensor as randomly as raindrops, and cannot be eliminated in principle. Let X be the number of photoelectrons accumulated in the sensor during the shutter aperture time, and X follows a Poisson distribution Po(λ) with population λ. Note that for $\lambda > 20$ or so, Po(λ) is very close to the normal distribution $N(\lambda,\lambda)$. λ is equal to both the expected value of X (i.e., $E(X)$) and the variance $V(X)$, which leads to $S/N=\sqrt{S}$, where S and N are the magnitudes of signal and noise, respectively. The signal-to-noise ratio due to optical shot noise becomes worse the darker the image, but the noise itself becomes smaller the darker the image.

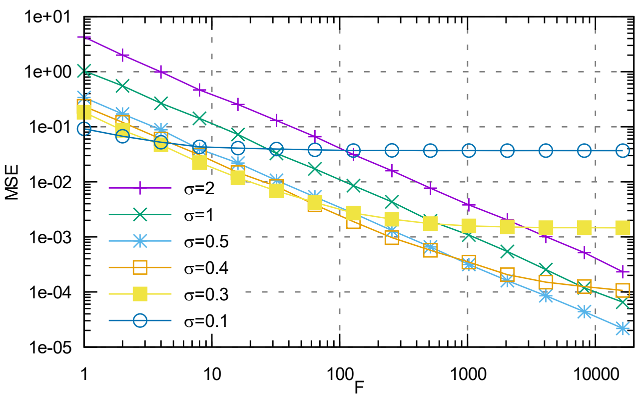

Fig. 1. Number of trials (F) vs. Remaining noise (in MSE) after additive averaging for each σ.

Consider the situation where the output value x increases by 1 for every N photoelectrons accumulated in the sensor (N corresponds to the full well capacity of the sensor divided by the maximum digital output value). Let $x=X/N$ and let the expected value $E(x)$ be the "true pixel value x_t". Then

$$x_t = E(x) = E(X)/N = \lambda/N$$

and

$$\sigma_x^2 = V(x) = V\frac{X}{N^2} = \frac{\lambda}{N^2}$$

leads to $\sigma_x = \sqrt{x_t/N}$. In real camera measurements, $\sigma_x = 0.17\sqrt{x_t}$ and $\sigma_x = \sqrt{x_t/53}$ have been observed[2,3], which correspond to $N = 34.6$ and 53, respectively. In the following discussion, $N = 53$ will be used.

2.3 Properties of the expected value of the additive averaging method

Digital imaging equipment outputs x_D, which is rounded and clipped value of $x = X/N$. Hereafter the output is assumed to be represented by 10 bits. The expected value of x_D is obtained by

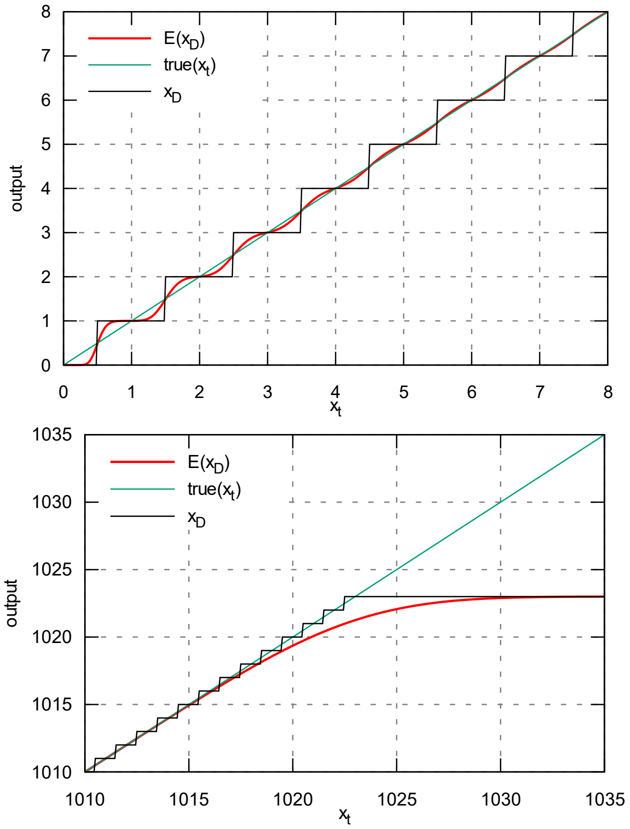

Fig. 2. True value (x_t) vs. expected value ($E(x_D)$) after digitizing. (top: dark area, bottom: light area)

using the probability mass function $f(\lambda, k)$ of the Poisson distribution with population λ as follows.

$$E(x_D) = \sum_{k=0}^{\infty} f(Nx_t, k) Digital(k/N)$$

where $Digital(a) = min(\lfloor a + 0.5 \rfloor, 1023)$ is the rounding and clipping function; $E(x_D)$ is monotonically increasing with respect to x_t. The relationship between the two is illustrated in Fig. 2. In dark areas, the optical shot noise is relatively small and x_D sticks to a single integer value, causing $E(x_D)$ to wave, which is the cause of the convergence phenomenon observed in Fig. 1 when $\sigma < 0.4$. In the light area, the increase in $E(x_D)$ slows down after the true value exceeds 1015 and approaches 1023 asymptotically. In the middle part $(10 < x_t < 1010)$, $E(x_D) \simeq x_t$.

3. Proposed method and simulation results

Once the correspondence between x_t and $E(x_D)$ is obtained as shown in Fig. 2, it becomes possible to bring the additive average value closer to the true value by the inverse transformation of above correspondence. This is referred to as the proposed correction method.

Fig. 3 shows the results of generating Poisson noise equivalent to optical shot noise, averaging the number of trials ($F = 100$), and performing the correction using the proposed method. In Fig. 3 top, it is observed that the average value is wavy like the theoretical value $E(x_D)$ in Fig. 2. The proposed correction value is close to the true value, and the estimated value exceeds the upper limit (1023) of the digital value (Fig. 3 bottom). When $F = 10{,}000$, we could sense up to 1037.

Table 1 shows the MSE comparison for the true values in the dark (0−8) and light (1010−1030) areas for some number of trials F. The MSE of the proposed method is 1/10 to 1/40 lower than

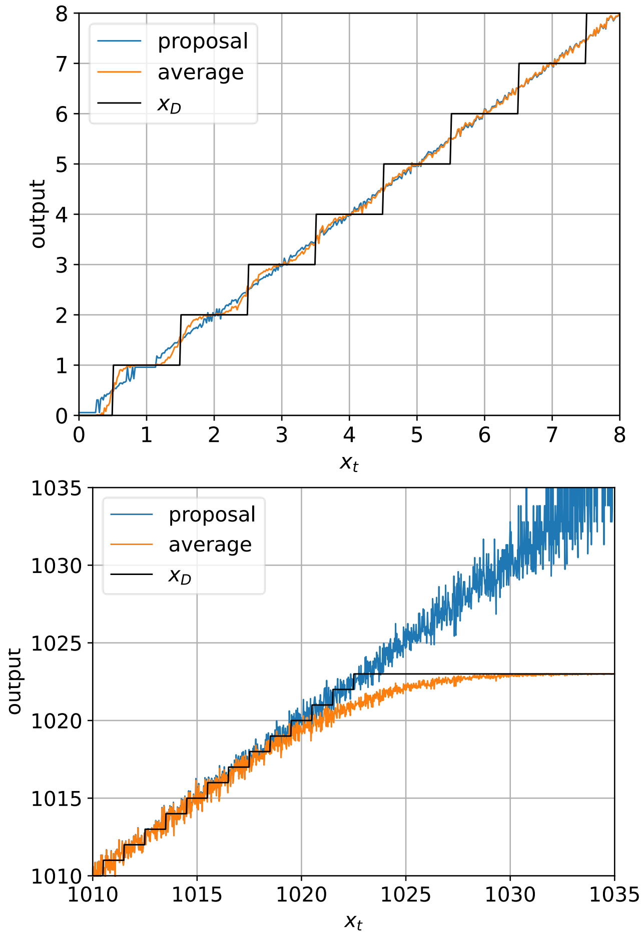

Fig. 3. 100 additions average and its correction value (proposal). (top: dark area, bottom: Light area)

Table 1. MSE (x100 for dark areas) comparison. (conv.: conventional additive averaging)

	$F = 100$		$F = 1{,}000$		$F = 10{,}000$	
area	conv.	proposal	conv.	proposal	conv.	proposal
dark	7.56	3.74	6.66	1.40	6.48	0.522
light	1.81	0.461	1.72	0.133	1.70	0.0442

that of the conventional method.

4. Conclusion

In this paper, we scrutinized the temporal brightness fluctuations that light necessarily has, discovered the problems with the additive averaging method and the way of correction, and revealed the possibility of sensing beyond the upper limit of measurement values.

References

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